Girraween High School

Year 12 Mathematics Extension 2

2026 HSC Task 3

Student Number _____

Question	Mark	MEX12-1	MEX12-2	MEX12-5	MEX12-7	MEX12-8
1	/1	✓		✓	✓	
2	/1	✓		✓	✓	
3	/1	✓		✓	✓	
4	/1	✓	✓		✓	
5	/1	✓	✓		✓	
6	/12	✓		✓	✓	✓
7	/15	✓		✓	✓	✓
8	/12	✓		✓	✓	✓
9	/9	✓	✓		✓	✓
10	/11	✓	✓		✓	✓
11	/9	✓	✓		✓	✓
Total	/73	/73	/31	/42	/73	/68

Year 12 Mathematics Extension 2 outcomes

A student:

MEX12-1 understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts

MEX12-2 chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings

MEX12-3 uses vectors to model and solve problems in two and three dimensions

MEX12-4 uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems

MEX12-5 applies techniques of integration to structured and unstructured problems

MEX12-6 uses mechanics to model and solve practical problems

MEX12-7 applies various mathematical techniques and concepts to model and solve structured, unstructured and multi-step problems

MEX12-8 communicates and justifies abstract ideas and relationships using appropriate language, notation and logical argument

2026 Year 12 Trial

Mathematics Extension 2

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ⬭ B ⬬ C ⬭ D ⬭

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ⬬ B ⬬ C ⬭ D ⬭

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

correct

A ⬬ B ⬬ C ⬭ D ⬭

1.	A	○	B	○	C	○	D ○
2.	A	○	B	○	C	○	D ○
3.	A	○	B	○	C	○	D ○
4.	A	○	B	○	C	○	D ○
5.	A	○	B	○	C	○	D ○

Girraween High School

2026 Year 12 HSC Task 3

Mathematics Extension 2

General Instructions

- Reading time: 5 minutes
- Working time: 90 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Answer multiple-choice questions on the multiple-choice answer sheet provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 73

Section I 5 Marks

- Attempt Questions 1 - 5
- Allow about 10 minutes for this section

Section II 68 Marks

- Attempt Questions 6 - 11
- Allow about 80 minutes for this section

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Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section
Use the multiple-choice answer sheet for Questions 1 – 5

Question 1

1. Which of the following is an expression for $\int \frac{1}{\sqrt{7-6x-x^2}}\,dx$?

A) $sin^{-1}\left(\frac{x-3}{2}\right)+c$

B) $sin^{-1}\left(\frac{x+3}{2}\right)+c$

C) $sin^{-1}\left(\frac{x-3}{4}\right)+c$

D) $sin^{-1}\left(\frac{x+3}{4}\right)+c$

Question 2

What is the value of $\int_0^1 \frac{e^x}{(1+e^x)^2}\,dx$?

A) $\frac{e-1}{2(e+1)}$

B) $\frac{e+1}{2(e-1)}$

C) $\frac{2(e-1)}{e+1}$

D) $\frac{2(e+1)}{e-1}$

Examination continues next page…

Question 3

Which of the following is an expression for $I_n = \int_1^e (log_e x)^n \ dx$, where $n = 1, 2, 3 \ldots$

A) $I_n = e - nI_{n-1}$

B) $I_n = 1 - nI_{n-1}$

C) $I_n = (e - 1) - nI_{n-1}$

D) $I_n = (e + 1) - nI_{n-1}$

Question 4

A lawyer for a person under investigation for a bank robbery states

"If my client was not in the suburb at the time of the robbery, the robber cannot be my client."

Which of the following statements is logically equivalent to the statement made by the lawyer?

A) If my client was in the suburb, then he was the robber.

B) If my client was the robber, then my client was in the suburb.

C) If my client was not the robber, then he was not in the suburb.

D) If my client was not the robber, then my client was in the suburb.

Question 5

Assume that a and b are negative real numbers with $a > b$. Which of the following might be false?

A) $\frac{1}{a-b} < 0$

B) $\frac{a}{b} - \frac{b}{a} < 0$

C) $a + b > 2b$

D) $2a > 3b$

Examination continues next page…

Section II

68 marks
Attempt Questions 6 - 11
Allow about 80 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (12 marks) **Start a new writing page**

a) Find $\int x\sqrt{2x-1}\ dx$. 3

b) Find $\int \frac{e^{2x}}{e^{x}+1}\ dx$ 2

c) Find $\int \tan^{-1} x\ dx$ 2

d) i) Find constants A, B and C such that $\frac{x^2+5x+2}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$. 3

ii) Hence find $\int \frac{x^2+5x+2}{(x^2+1)(x+1)}\ dx$. 2

Examination continues next page…

Question 7 (15 marks) **Start a new writing page**

a) Find $\int \frac{1}{x^2-12x+52}\, dx$ 2

b) Find $\int \frac{x^2}{\sqrt{25-x^2}}\, dx.$ 3

c) i) Use the substitution $t = \tan\frac{x}{2}$ to show that $\int_0^{\frac{\pi}{2}} \frac{dx}{(1+\frac{1}{2}\sin x)} = \frac{2\pi}{3\sqrt{3}}$ 4

ii) Show that $\int_0^{2a} f(x)\ dx = \int_0^a [f(x) + f(2a-x)\ ]\, dx$ 3

iii) Hence evaluate $\int_0^{\pi} \frac{x}{(1+\frac{1}{2}\sin x)}\, dx$ 3

Examination continues next page…

Question 8 (12 marks) **Start a new writing page**

a) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-cosx}{1+cosx}}\, dx$ 3

b) Find $\int \sin^5\theta \cos^4\theta \ \ d\theta$ 3

c) i) Show that $(1-\sqrt{x})^{n-1}\sqrt{x} = (1-\sqrt{x})^{n-1} - (1-\sqrt{x})^n$ 1

ii) If $I_n = \int_0^1 (1-\sqrt{x})^n\, dx$ for $n \geq 0$, show that $I_n = \frac{n}{n+2} I_{n-1}$ for $n \geq 1$ 3

iii) Deduce that $\frac{1}{I_n} = \binom{n+2}{n}$ for $n \geq 2$ 2

Examination continues next page…

Question 9 (9 marks) **Start a new writing page**

a) Prove by Mathematical Induction that $\frac{1}{n!} < \frac{1}{2^{n-1}}$ for $n \geq 3$, $n \in \mathbb{Z}^+$. 3

b) A sequence is defined recursively by $t_{n+1} = \frac{t_n}{t_n + 1}$ for integers $n \geq 1$, where $t_1 = 2$. 3

Prove by Mathematical Induction that $t_n = \frac{2}{2n-1}$ for all positive integers n.

c) Prove by contradiction that if *n* is a positive integer then $\sqrt{8n + 6}$ is always irrational. 3

Examination continues next page…

Question 10 (11 marks) **Start a new writing page**

a) If m and n are odd, prove $m^3 + n^3$ is even. 2

b) Consider the statement P: "If n^3 is a multiple of 3, then n is a multiple of 3."

i) Write down the contrapositive of P. 1

ii) Prove the statement P by contraposition. 2

c) Given that $x + y = m$ with $x > 0$ and $y > 0$

i) Show that $(x + y)^2 \geq 4xy$ 2

ii) Hence prove that $\frac{1}{x} + \frac{1}{y} \geq \frac{4}{m}$ 2

iii) Given the AM-GM inequality $\frac{a+b}{2} \geq \sqrt{ab}$ and using the result from part ii), prove that $\frac{1}{x^2} + \frac{1}{y^2} \geq \frac{8}{m^2}$ 2

Examination continues next page…

Question 11 (9 marks) **Start a new writing page**

a) i) If $a > 0$ is a real number, show that $a + \frac{1}{a} \geq 2$ 1

ii) Hence show that if $a > 0, b > 0, c > 0$ are real numbers, then

α) $\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c} \geq 6$ 2

β) $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$ 2

b) i) If a and b are positive real numbers, show that $a^2 - ab + b^2 \geq \left(\frac{a+b}{2}\right)^2$. 1

ii) In ΔABC, if $\angle BCA \geq 60^o$ show that $c^2 \geq a^2 - ab + b^2$ and hence deduce that $2c^3 \geq a^3 + b^3$ with equality, if and only if ΔABC is equilateral. 3

End of Examination

Mathematics Assessment Solutions

Girraween High School

☐ Year 11	☐ Extension 1	Year 2026
☑ Year 12	☑ Extension 2	Task Number 3

Suggested solutions	Marker's Feedback

multiple choice

Q1 $\int \frac{1}{\sqrt{7-6x-x^2}}\,dx$

$= \int \frac{1}{\sqrt{7-9+9-6x-x^2}}\,dx$

$= \int \frac{1}{\sqrt{16-(9+6x+x^2)}}\,dx$

$= \int \frac{1}{\sqrt{16-(3+x)^2}}\,dx$

$= \sin^{-1}\left(\frac{x+3}{4}\right) + C$ (D)

Q2 Let $u = 1+e^x$

$du = e^x\,dx$

$x = 1 \rightarrow u = 1+e$

$x = 0 \rightarrow u = 2$

$\int_0^1 \frac{e^x}{(1+e^x)^2}\,dx = \int_2^{1+e} \frac{du}{u^2}$

ctd

Suggested solutions	Marker's Feedback

$$= \int_2^{1+e} u^{-2}\, du$$

$$= \left[-u^{-1}\right]_2^{1+e}$$

$$= \frac{-1}{1+e} - \frac{-1}{2}$$

$$= \frac{e-1}{2(1+e)}$$

(A)

Q3 $I_n = \int_1^e (\log_e x)^n\, dx$

$u = \log_e x$ $\quad v = x$

$du = n \cdot \frac{1}{x} (\log_e x)^{n-1}$ $\quad dv = 1$

$$I_n = \left[x \cdot \log_e x\right]_1^e - n \int_1^e \frac{1}{x} (\log_e x)^{n-1} \cdot 1 \, dx$$

$$= e - n I_{n-1}$$

(A)

Q4 The contrapositive is the logical equivalent statement of the form $\sim p \to \sim q$. Contrapositive is $q \to p$.

(B)

Suggested solutions	Marker's Feedback
Q5 (A) is the only one that is false $a > b$ $a - b > 0$ $\frac{1}{a-b} < 0$ which is false.	

Suggested solutions	Marker's Feedback

Q 6

a) $\int x\sqrt{2x-1}\,dx$ let $u = 2x-1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$= \int \frac{u+1}{2}\sqrt{u}\,\frac{1}{2}\,du$ ① change of variable

$= \frac{1}{4}\int (u+1)u^{\frac{1}{2}}\,du$

$= \frac{1}{4}\int (u^{\frac{3}{2}} + u^{\frac{1}{2}})\,du$

$= \frac{1}{4}\left(\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right) + C$ ① integrating

$= \frac{1}{2}\left(\frac{\sqrt{(2x-1)^5}}{5} + \frac{\sqrt{(2x-1)^3}}{3}\right) + C$ ① writing in terms of x

or equivalent

Generally well done. A lot of different methods were used (see next page)

Alternate solutions to (6)(a)

Q.(6)(a)	Suggested solutions	Marker's Feedback

$\int x\sqrt{2x-1}\,.dx$ By parts $u = x$ $v = \frac{1}{3}(2x-1)^{\frac{3}{2}}$

$u' = 1$ $v' = \sqrt{2x-1}$

By $\int u\,v'.dx = uv - \int v u'.dx$

$\int x\sqrt{2x-1}\,.dx = \frac{1}{3}x(2x-1)^{\frac{3}{2}} - \frac{1}{3}\int (2x-1)^{\frac{3}{2}}.dx$

$= \frac{1}{3}x(2x-1)^{\frac{3}{2}} - \frac{1}{15}(2x-1)^{\frac{5}{2}} + C$ → Full mks were paid here.

$=$ * when simplified using $x = \frac{(2x-1)+1}{2}$

$= \frac{(2x-1)^{\frac{5}{2}}}{10} + \frac{1}{6}(2x-1)^{\frac{3}{2}} + C$

(or)

$\int x\sqrt{2x-1}\,.dx$ $x = \frac{1}{2}\sec^2\theta\, dx = \tan\theta\sec^2\theta.d\theta$

$2x-1 = \tan^2\theta$

$= \int \frac{1}{2}\sec^2\theta.\tan\theta.\tan\theta\sec^2\theta.d\theta$

$= \frac{1}{2}\int (\tan^2\theta + \tan^4\theta).\sec^2\theta.d\theta$

$= \frac{1}{6}\tan^3\theta + \frac{1}{10}\tan^5\theta + C$

$= \frac{1}{6}(2x-1)^{\frac{3}{2}} + \frac{1}{10}(2x-1)^{\frac{5}{2}} + C.$

Alternative (6)(b)

$\int \frac{e^{2x}}{e^x+1}\,.dx$ $u = e^x+1.\ du = e^x.dx$

$= \int \frac{e^x}{e^x+1}.e^x.dx$

$= \int \frac{u-1}{u}.du$

$= u - \ln(u) + C$

$= e^x+1 - \ln(e^x+1) + C$

1

Suggested solutions	Marker's Feedback

Q6 5) $\int \frac{e^{2x}}{e^x+1}\,dx$

$= \int \frac{e^x(e^x+1)-e^x}{e^x+1}\,dx$

$= \int e^x - \frac{e^x}{e^x+1}\,dx$

$= e^x - \ln(e^x+1) + C$

Generally well done.
Some students tried to use $\int \frac{e^{2x}-1}{e^x+1}\,dx + \int \frac{1}{e^x+1}\,dx$ ① progress

$= \int e^x - 1 + \frac{1}{e^x+1}\,dx$

which got no mark as no progress was made.
Some students did ① solution manage to do

$= \int e^x - 1 + \frac{e^{-x}}{e^{-x}+1}\,dx$

$= e^x - x + \ln(e^{-x}+1) + C$

which got full mark.

c) $\int \tan^{-1} x\,dx$

$u = \tan^{-1} x \qquad v = x$

$du = \frac{1}{1+x^2} \qquad dv = 1$

$= x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx$

$= \tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$

Very well done.
Alternative solution on p.40

① applies integration by parts ① solution

Suggested solutions	Marker's Feedback

Q6 d) i)

$$\frac{x^2+5x+2}{(x+1)(x^2+1)} \equiv \frac{Ax+b}{x^2+1} + \frac{C}{x+1}$$

$x^2+5x+2 \equiv (Ax+b)(x+1) + C(x^2+1)$

let $x=-1$

$-2 = 2C$

$\therefore C = -1$

let $x=0$

$2 = b.1 + C$

$2 = b-1$

$\therefore b = 3$

equating coefficients of x^2

$1 = A + C$

$1 = A + -1$

$\therefore A = 2$

① demonstrates understanding of process

① one correct value

Very well done.

① all three correct values

ii) $\int \frac{2x+3}{x^2+1} - \frac{1}{x+1}\, dx$ Very well done.

$= \int \frac{2x}{x^2+1} + \frac{3}{x^2+1} - \frac{1}{x+1}\, dx$

$= \ln(x^2+1) + 3\tan^{-1}x - \ln(x+1) + C$

① one correct integral

① all correct.

Suggested solutions	Marker's Feedback

Q7

a) $\int \frac{1}{x^2-12x+52}\,dx$

$= \int \frac{1}{x^2-12x+36-52-36}\,dx$

$= \frac{1}{(x-6)^2+16}\,dx$ ①

done well

$= \frac{1}{4}\tan^{-1}\left(\frac{x-6}{4}\right)+C$ ①

b)

$I = \int \frac{x^2}{\sqrt{25-x^2}}\,dx$ let $x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta$

$= \int \frac{25\sin^2\theta}{\sqrt{25-25\sin^2\theta}} \cdot 5\cos\theta\,d\theta$ ① change of variable

$= \int \frac{25\sin^2\theta\ 5\cos\theta\,d\theta}{5\cos\theta}$

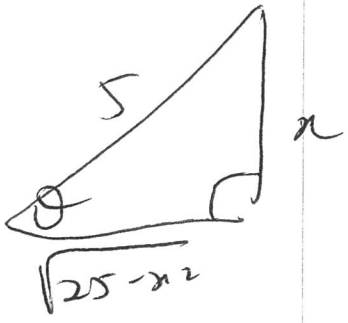
$= \int 25\sin^2\theta\,d\theta$

$= 25\int \frac{1}{2}(1-\cos 2\theta)\,d\theta$

Suggested solutions	Marker's Feedback

$$I = \frac{25}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$ ① integrating

Now, $\sin\theta = \frac{x}{5}$



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2 \cdot \frac{x}{5} \cdot \frac{\sqrt{25-x^2}}{5}$$
$$= \frac{2}{25} x\sqrt{25-x^2}$$

$$I = \frac{25}{2}\left[\sin^{-1}\frac{x}{5} - \frac{1}{2}\cdot\frac{2}{25} x\sqrt{25-x^2}\right] + C$$

$$= \frac{25}{2}\sin^{-1}\frac{x}{5} - \frac{x}{2}\sqrt{25-x^2} + C$$ ① in terms of x.

The best way for doing this question is - use the substitution $x = 5\sin\theta$

Some students split the question as two integrals. Then used $x = 5\sin\theta$ to integrate one integral. It is a waste of time and also making the question ~~hard~~ hard.

Suggested solutions	Marker's Feedback

c) $\int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{1}{2}\sin x}\,dx$ (1)

$= \int_0^1 \frac{1}{1+\frac{1}{2}\cdot\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt$

$= \int_0^1 \frac{1}{\frac{1+t+t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt$

$= \int_0^1 \frac{2}{t^2+t+1}\,dt$

$= \int_0^1 \frac{2}{(t+\frac{1}{2})^2+\frac{3}{4}}\,dt$ (1)

$= 2\left[\frac{2}{\sqrt{3}} \tan^{-1}\frac{(t+\frac{1}{2})}{\frac{\sqrt{3}}{2}}\right]_0^1$

$= \frac{4}{\sqrt{3}}\left[\tan^{-1}\frac{3}{2}\cdot\frac{2}{\sqrt{3}} - \tan^{-1}\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\right]$

$= \frac{4}{\sqrt{3}}\left(\tan^{-1}\frac{3}{\sqrt{3}} - \tan^{-1}\frac{1}{\sqrt{3}}\right)$

$= \frac{4}{\sqrt{3}}\cdot\frac{\pi}{6}$

$= \frac{2\pi}{3\sqrt{3}}$

Let $t = \tan\frac{x}{2}$

$\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$

$\frac{dt}{dx} = \frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)$

$dt = \frac{1}{2}(1+t^2)\,dx$ (1)

$dx = \frac{2}{1+t^2}\,dt$

$x = \frac{\pi}{2} \rightarrow t = 1$

$x = 0 \rightarrow t = 0$

Some students made mistake in completing the square.

(1)

Suggested solutions	Marker's Feedback

ii) $\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$ (1)

For $\int_a^{2a} f(x)\,dx$ let $x = 2a - t$

$dx = -dt$

$x = 2a \rightarrow t = 0$

$x = a \rightarrow t = a$

This is a difficult one for many students.

$= \int_0^{a} f(x)\,dx + \int_a^{0} f(2a-t)\cdot -dt$ (1)

$= \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-t)\,dt$

$= \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$ (1)

$= \int_0^{a} [f(x) + f(2a-x)]\,dx$

iii) $\int_0^{\pi} \frac{x}{(1+\frac{1}{2}\sin x)}\,dx$

$= \int_0^{\frac{\pi}{2}} \frac{x}{1+\frac{1}{2}\sin x} + \frac{\pi - x}{1+\frac{1}{2}\sin(\pi - x)}\,dx$ (1)

$= \int_0^{\frac{\pi}{2}} \frac{x}{1+\frac{1}{2}\sin x} + \frac{\pi - x}{1+\frac{1}{2}\sin x}\,dx$

Suggested solutions	Marker's Feedback

$$= \int_0^{\frac{\pi}{2}} \frac{x + \pi - x}{1 + \frac{1}{2}\sin x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\pi}{1 + \frac{1}{2}\sin x}\, dx \quad \text{①}$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{1}{1 + \frac{1}{2}\sin x}\, dx$$

Most did well.

$$= \pi \times \frac{2\pi}{3\sqrt{3}} \quad \text{from part (i)}$$

$$= \frac{2\pi^2}{3\sqrt{3}} \quad \text{①}$$

Suggested solutions	Marker's Feedback

See alternate solution

Q8

a) $\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}}\, dx \propto \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$

$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos^2 x}{(1+\cos x)^2}}\, dx$

↑ by multiplying the conjugate of the numerator

$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin^2 x}{(1+\cos x)^2}}\, dx$

$= \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}\, dx$ ① simplifying

$= -\int_0^{\frac{\pi}{2}} \frac{-\sin x}{1+\cos x}\, dx$

$= -\left[\ln(1+\cos x)\right]_0^{\frac{\pi}{2}}$ ① integrating

$= -\left[\ln\left(1+\cos\frac{\pi}{2}\right) - \ln(1+\cos 0)\right]$

$= -\ln 1 + \ln 2$

$= \ln 2$ ① evaluating

many different methods were used. Students who used this method were the most successful in getting the correct answer.

Suggested solutions	Marker's Feedback

(8a) alternate solution

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1+\cos x}{1-\cos x}}\,dx \propto \frac{\sqrt{1-\cos x}}{\sqrt{1-\cos x}}$$

very popular method

$$= \int_0^{\frac{\pi}{2}} \frac{1-\cos x}{\sqrt{1-\cos^2 x}}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1-\cos x}{\sin x}\,dx \quad (1)$$

$$= \int_0^{\frac{\pi}{2}} \operatorname{cosec} x - \cot x\,dx$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{\operatorname{cosec} x(\operatorname{cosec} x + \cot x)}{\operatorname{cosec} x + \cot x}\right] - \frac{\cos x}{\sin x}\,dx$$

$$= \Big[-\ln|\operatorname{cosec} x + \cot x| - \ln|\sin x|\Big]_0^{\frac{\pi}{2}} \quad (1)$$

must combine

$$= -\left[\ln\left|\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right| + \ln|\sin x|\right]_0^{\frac{\pi}{2}}$$

$$= -\Big[\ln|1+\cos x|\Big]_0^{\frac{\pi}{2}}$$

$$= -(\ln 1 - \ln 2)$$

$$= \ln 2 \quad (1)$$

students who did not combine would have found $\ln|\sin x|$

Suggested solutions	Marker's Feedback

Q8 a) alternate solution

$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos x}{1+\cos x}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-(1-2\sin^2\frac{x}{2})}{1+(2\cos^2\frac{x}{2}-1)}}$$ ①

using identity
$\cos 2\theta = 2\cos^2\theta - 1$
$\cos 2\theta = 1 - 2\sin^2\theta$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\tan^2\frac{x}{2}}\, dx$$

not a commonly used method but efficient

$$= \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}\, dx$$

$$= -2\left[\ln\left|\cos\frac{x}{2}\right|\right]_0^{\frac{\pi}{2}}$$ ①

$$= -2\left[\ln\cos\frac{\pi}{4} - \ln 1\right]$$

$$= -2\ln\left(\frac{1}{\sqrt{2}}\right)$$

$$= 2\ln\sqrt{2}$$

$$= \ln 2$$ ①

Suggested solutions	Marker's Feedback

b) $\int \sin^5\theta \cos^4\theta \, d\theta$

Mostly well done

let $u = \cos\theta$

$du = -\sin\theta \, d\theta$

$= \int \sin^4\theta \cdot \cos^4\theta \sin\theta \, d\theta$

$= -\int (1-u^2)^2 \cdot u^4 \, du$ ① change of variable

$= -\int (u^4 - 2u^6 + u^8) \, du$

$= -\frac{u^5}{5} + \frac{2u^7}{7} - \frac{u^9}{9} + C$ ① integrating

$= -\frac{1}{5}\cos^5\theta + \frac{2}{7}\cos^7\theta - \frac{1}{9}\cos^9\theta + C$ ① in terms of θ.

Some students did not return their solution to in terms of θ.

c) LHS $= (1-\sqrt{x})^{n-1}\sqrt{x}$

well done.

$= (1-\sqrt{x})^{n-1}(\sqrt{x}-1+1)$

$= (1-\sqrt{x})^{n-1}(\sqrt{x}-1) + (1-\sqrt{x})^{n-1}$

$= (1-\sqrt{x})^{n-1} - (1-\sqrt{x})(1-\sqrt{x})^{n-1}$ } ①

$= (1-\sqrt{x})^{n-1} - (1-\sqrt{x})^n$ = RHS

Alternate solution

RHS $= (1-\sqrt{x})^{n-1} - (1-\sqrt{x})^n$

$= (1-\sqrt{x})^{n-1}(1-(1-\sqrt{x}))^n$

$= (1-\sqrt{x})^{n-1}\sqrt{x}$

= LHS

Suggested solutions	Marker's Feedback

See alternate solution

ii) $I_n = \int_0^1 (1-\sqrt{x})^n \, dx$ from part (i)

$= \int_0^1 (1-\sqrt{x})^{n-1} - (1-\sqrt{x})^{n-1}\sqrt{x}\, dx$ ①

$= I_{n-1} - \int_0^1 \sqrt{x}(1-\sqrt{x})^{n-1} dx$

$= I_{n-1} + \int_0^1 \frac{2n}{-2\sqrt{x}} (1-\sqrt{x})^{n-1} dx$

$u = 2n \qquad v = \frac{(1-\sqrt{x})^n}{n}$

$du = 2 \qquad dv = \frac{-1}{2\sqrt{x}}(1-\sqrt{x})^{n-1}$ } ①

$I_n = I_{n-1} + \left[\frac{2x(1-\sqrt{x})^n}{n}\right]_0^1 - \frac{2}{n}\int_0^1 (1-\sqrt{x})^n \, dx$

$I_n = I_{n-1} - \frac{2}{n} I_n$ } ①

$(n+2) I_n = n I_{n-1}$

$I_n = \frac{n}{n+2} I_{n-1}$

Generally well done.

Suggested solutions	Marker's Feedback

alternate solution

8 c) ii)

$$I_n = \int_0^1 (1 - x^{\frac{1}{2}})^n \, dx$$

let $u = (1 - x^{\frac{1}{2}})^n$ $\quad v' = 1$

$u' = \frac{-n}{2\sqrt{x}}(1 - x^{\frac{1}{2}})^{n-1}$ $\quad v = x$ } (1)

$$I_n = \left[x(1 - \sqrt{x})^n \right]_0^1 - -\int_0^1 \frac{nx}{2\sqrt{x}} (1 - \sqrt{x})^{n-1} \, dx$$

$$= 0 + \frac{n}{2} \int_0^1 \sqrt{x} (1 - x)^{n-1} \, dx$$

$$= \frac{n}{2} \int_0^1 (1 - \sqrt{x})^{n-1} - (1 - \sqrt{x})^n \, dx \quad \text{from part (i)} \quad (1)$$

$$= \frac{n}{2} \int_0^1 (1 - \sqrt{x})^{n-1} \, dx - \frac{n}{2} \int_0^1 (1 - \sqrt{x})^n \, dx$$

$$I_n = \frac{n}{2} I_{n-1} - \frac{n}{2} I_n$$

$$2 I_n = n I_{n-1} - n I_n$$

$$(2 + n) I_n = n I_{n-1}$$ } (1)

$$I_n = \frac{n}{n+2} I_{n-1} \quad \text{as required}$$

Suggested solutions	Marker's Feedback

(iii) $I_n = \frac{n}{n+2} \cdot \frac{n-1}{n+1} \cdot I_{n-2}$

$= \frac{n}{n+2} \cdot \frac{n-1}{n+1} \cdot \frac{n-2}{n} \cdot I_{n-3}$

$= \frac{n}{n+2} \cdot \frac{n-1}{n+1} \cdots \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} I_0 \times \frac{2}{2}$ (1)

$I_0 = \int_0^1 dx = [x]_0^1 = 1$

justification needed.

(1)

$I_n = \frac{2n!}{(n+2)!}$

$\frac{1}{I_n} = \frac{(n+2)!}{2!\,n!}$

$\frac{1}{I_n} = \binom{n+2}{n} \quad n \geq 2.$

Sufficient working or reasoning required to achieve full marks.

Suggested solutions	Marker's Feedback

Q9 a) $\frac{1}{n!} < \frac{1}{2^{n-1}}$ $n \geq 3$

i) prove true for $n=3$

LHS $= \frac{1}{3!} = \frac{1}{6}$

RHS $= \frac{1}{2^{3-1}} = \frac{1}{4}$

LHS < RHS

$\therefore$ true for $n=3$ ①

ii) Assume true for $n=k$

$\frac{1}{k!} < \frac{1}{2^{k-1}}$

prove true for $n=k+1$

$\frac{1}{(k+1)!} < \frac{1}{2^k}$

LHS $= \frac{1}{(k+1)!}$

$= \frac{1}{k!(k+1)}$

$< \frac{1}{2^{k-1}} \cdot \frac{1}{k+1}$ from assumption

$= \frac{1}{k \cdot 2^{k-1} + 2^{k-1}}$

$< \frac{1}{2 \cdot 2^{k-1} + 2^{k-1}}$ since $k \geq 3$

①

Generally well done

Different methods used.

$$= \frac{1}{3 \cdot 2^{k-1}}$$

$$< \frac{1}{2 \cdot 2^{k-1}}$$

$$= \frac{1}{2^k}$$

$$\therefore \frac{1}{(k+1)!} < \frac{1}{2^k}$$ } (1)

If true for $n=k$, it is also true for $n=k+1$.

Hence by the principle of mathematical induction the statement is true for $n \geq 3$

Alternate solution

from assumption

$$\frac{1}{k!} < \frac{1}{2^{k-1}}$$

multiply by $\frac{1}{k+1}$

$$\frac{1}{k!(k+1)} < \frac{1}{2^{k-1}} \cdot \frac{1}{k+1}$$ } (1)

Suggested solutions	Marker's Feedback

$$\frac{1}{(k+1)!} < \frac{1}{2^{k+1}(k+1)} \times \frac{2}{2}$$

$$< \frac{1}{2^k} \cdot \frac{2}{k+1}$$

$$\frac{1}{(k+1)!} < \frac{1}{2^k} \quad \text{since} \quad 0 < \frac{2}{k+1} < 1$$

for $k > 3$

conclusion

Prove by Mathematical induction that

$$\frac{1}{n!} < \frac{1}{2^{n-1}} \quad \text{for } n \geq 3,\ n \in \mathbb{Z}^+$$

Alternate solution

Step 1 $n = 3$

$$\text{LHS} = \frac{1}{n!} = \frac{1}{3!} = \frac{1}{6}$$

$$\text{RHS} = \frac{1}{2^{n-1}} = \frac{1}{2^{3-1}} = \frac{1}{2^2} = \frac{1}{4}$$

$$\frac{1}{6} < \frac{1}{4} \quad \therefore \text{ true for } n = 3$$

Step 2

Assume the result is true for $n = k$, where $k \geq 3$

$$\frac{1}{k!} < \frac{1}{2^{k-1}} \quad \text{— (1)}$$

To prove for $n = k+1$

$$\frac{1}{(k+1)!} < \frac{1}{2^{k+1-1}}$$

$$\frac{1}{(k+1)!} < \frac{1}{2^k} \quad \text{— (2)}$$

Step 3

LHS of (2) $= \dfrac{1}{(k+1)!}$

$$= \frac{1}{k!(k+1)}$$

$$= \frac{1}{k!} \times \frac{1}{k+1}$$

$$< \frac{1}{2^{k-1}} \times \frac{1}{k+1} \quad \left(\text{by assumption } ①\right)$$

$$= \frac{1}{2^{k-1}} \times \frac{2}{2} \times \frac{1}{(k+1)}$$

$$= \frac{1}{2^k} \times \frac{2}{k+1}$$

$$< \frac{1}{2^k}$$

$$\left(\text{since } \frac{2}{k+1} < 1 \text{ for } k \geq 3\right.$$

$$k=3, \ \frac{2}{k+1} = \frac{2}{4} = \frac{1}{2}$$

$$\left. k=4, \ \frac{2}{k+1} = \frac{2}{5}\right)$$

$$= \text{RHS of } ②$$

If the result is true for $n = k$ then it is true for $n = k+1$.

Hence by the principle of mathematical induction the result is true for all $n \geq 3$

Suggested solutions	Marker's Feedback

Q9 b).

i) prove true for $n=1$

$LHS = t_1 = 2$

$RHS = \frac{2}{2\times 1-1} = 2$

$LHS = RHS$

$\therefore$ true for $n=1$

Some students made no mention of the LHS. (1)

marks not deducted this time.

ii) Assume true for $n=k$

ie $t_k = \frac{2}{2k-1}$

Very well done.

iii) prove true for $n=k+1$

$t_{k+1} = \frac{2}{2(k+1)-1} = \frac{2}{2k+1}$ (1)

$LHS = t_{k+1}$

$= \frac{t_k}{t_{k+1}}$ recurrence formula

$= \frac{\frac{2}{2k-1}}{\frac{2}{2k-1}+1}$ from assumption

Suggested solutions	Marker's Feedback

$$= \frac{2}{\frac{2k-1}{2+2k-1}}$$

$$= \frac{2}{2k+1}$$

= RHS

(1)

$\therefore$ If true for $n=k$, then also true for $n=k+1$.

$\therefore$ Hence by the principle of mathematical induction, statement is true for $n \geq 1$.

Suggested solutions	Marker's Feedback

9(c) Assume $\sqrt{8n+6}$ is rational

i.e. $\sqrt{8n+6} = \frac{p}{q}$ where p, q are integers with no common factors except for 1, $q \neq 0$ (1)

$8n+6 = \frac{p^2}{q^2}$

$2(4n+3)q^2 = p^2$ (1)

p^2 is divisible by 2

$\therefore$ p is divisible by 2

let $p = 2k$

$(2k)^2 = 2(4n+3)q^2$

$4k^2 = 2(4n+3)q^2$

$2k^2 = (4n+3)q^2$

many students did not justify why q^2 is even

LHS is even, $(4n+3)$ is odd

$\therefore$ q^2 must be even (1)

Since q^2 is even,

q must be even

As both p, q are divisible by 2, this is a contradiction

$\therefore$ $\sqrt{8n+6}$ is irrational.

Q10

a) If m and n are odd

let $m = 2k+1$ $n = 2j+1$ } ① sets up proof

$m^3 + n^3 = (2k+1)^3 + (2j+1)^3$

$= 8k^3 + 12k^2 + 6k + 1$

$+ 8j^3 + 12j^2 + 6j + 1$

$= 2(4k^3 + 6k^2 + 3k$ } ①

$+ 4j^3 + 6j^2 + 3j + 1)$

$= 2p$ where p is an integer since k, j are integers

∴ If m, n are odd

then $m^3 + n^3$ is even

done well

Suggested solutions	Marker's Feedback

(b) i) Contrapositive: ①

If n is not a multiple of 3, then n^3 is not a multiple of 3.

All got correct.

ii) If n is not a multiple of 3, then $n = 3k \pm 1$ for integer k ①

$$\begin{aligned} n^3 &= (3k \pm 1)^3 \\ &= (3k)^3 \pm 3(3k)^2 + 3(3k) \pm 1^3 \\ &= 3(9k^3 \pm 9k^2 + 3k) \pm 1 \\ &= 3l \pm 1 \end{aligned}$$ ①

$\therefore n^3$ is not a multiple of 3

$\therefore$ If n is not a multiple of 3, then n^3 is not a multiple of 3.

Hence by contraposition, statement P is true.

Some students showed only one case. They lost 1 mark

Suggested solutions	Marker's Feedback

Q10 c) (i) $(x-y)^2 \geq 0$ (1)

$x^2 - 2xy + y^2 \geq 0$

$x^2 + 2xy + y^2 \geq 4xy$ (1)

$(x+y)^2 \geq 4xy$

done well.

(ii) $(x+y)^2 \geq 4xy$ (see alternate solution p24)

$x, y > 0$ (1)

$x+y \geq 0$

$xy > 0$

Some struggled to get it.

$\frac{x+y}{xy} \geq \frac{4}{x+y}$

$\frac{x}{xy} + \frac{y}{xy} \geq \frac{4}{x+y}$ (1)

$\frac{1}{x} + \frac{1}{y} \geq \frac{4}{x+y}$ as required.

(ii) AM – GM inequality

$\frac{a+b}{2} \geq \sqrt{ab}$

let $a = \frac{1}{x^2}$ $b = \frac{1}{y^2}$

Suggested solutions	Marker's Feedback

Q 1(b)(c) ii) alternate solution

$$\text{LHS} = \frac{1}{x} + \frac{1}{y}$$

$$= \frac{x+y}{xy}$$

$$= \frac{4(x+y)}{4xy}$$

$$\geq \frac{4(x+y)}{(x+y)^2} \quad \text{as } (x+y)^2 \geq 4xy \text{ from (i)}$$

$$= \frac{4}{x+y}$$

$$= \frac{4}{m}$$

$$= \text{RHS}$$

This was not an easy question for many students.

Suggested solutions	Marker's Feedback

$$\frac{\frac{1}{x^2}+\frac{1}{y^2}}{2} \geq \sqrt{\frac{1}{x^2}\cdot\frac{1}{y^2}}$$

$$\frac{1}{x^2}+\frac{1}{y^2} \geq 2\sqrt{\frac{1}{x^2y^2}}$$

$$\frac{1}{x^2}+\frac{1}{y^2} \geq \frac{2}{xy} \qquad xy>0 \qquad (1)$$

Now $\frac{1}{x^2}+\frac{1}{y^2}+\frac{2}{xy} = \left(\frac{1}{x}+\frac{1}{y}\right)^2$

$\geq \frac{16}{M^2}$ from (1) (1)

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{x^2}+\frac{1}{y^2} \geq \frac{16}{M^2}$$

$$2\left(\frac{1}{x^2}+\frac{1}{y^2}\right) \geq \frac{16}{M^2}$$

$$\frac{1}{x^2}+\frac{1}{y^2} \geq \frac{8}{M^2}$$

see alternate solution next page

Suggested solutions	Marker's Feedback

Q 10 c) iii) alternate solution

$(x+y)^2 \geq 4xy$ from (i) (1)

$\frac{x^2+y^2}{2} \geq \sqrt{x^2y^2}$ AM-GM inequality

$x^2+y^2 \geq 2xy$ (2)

multiply (1) and (2) $x, y > 0$

$(x+y)^2(x^2+y^2) \geq 4xy \cdot 2xy$ (1)

$m^2(x^2+y^2) \geq 8x^2y^2$

$\frac{x^2+y^2}{x^2y^2} \geq \frac{8}{m^2}$ $m^2 > 0$, $x^2y^2 > 0$ (1)

$\frac{x^2}{x^2y^2} + \frac{y^2}{x^2y^2} \geq \frac{8}{m^2}$

$\frac{1}{y^2} + \frac{1}{x^2} \geq \frac{8}{m^2}$

Suggested solutions	Marker's Feedback

Q11a)

i) $\left(a+\frac{1}{a}\right)^2 = \left(a-\frac{1}{a}\right)^2 + 4 \geq 4$

since $\left(a-\frac{1}{a}\right)^2 \geq 0$ (1)

Very well done, but most students used $\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)^2 \geq 0$

$\left(a+\frac{1}{a}\right)^2 \geq 4$

$a+\frac{1}{a} \geq 2$ for $a>0$

ii)

$LHS = \frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c}$

$= \left(\frac{a}{b}+\frac{b}{a}\right) + \left(\frac{b}{c}+\frac{c}{b}\right) + \left(\frac{c}{a}+\frac{a}{c}\right)$ (1) Generally well done.

$\geq 2 + 2 + 2$ using (i) (1)

$\therefore \frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c} \geq 6$

(ii) replace a with $b+c$

b with $c+a$

c with $a+b$ (1)

$\frac{(c+a)+(a+b)}{b+c} + \frac{(a+b)+(b+c)}{c+a} + \frac{(b+c)+(c+a)}{a+b}$

≥ 6

Suggested solutions	Marker's Feedback

$$1+\frac{2a}{b+c}+1+\frac{2b}{c+a}+1+\frac{2c}{a+b}\geq 6$$

$$2\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)\geq 3 \quad (1)$$

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\geq \frac{3}{2}$$

Very poorly done.

One correct solution which was seen was:

$$(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

$$=\left(\frac{x}{y}+\frac{y}{x}\right)+\left(\frac{y}{z}+\frac{z}{y}\right)+\left(\frac{x}{z}+\frac{z}{x}\right)+3$$

$$\geq 2+2+2+3$$

$$=9 \quad (1)$$

→ You HAD to show this part or you lost a mark.

then

$$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}$$

$$=\left(\frac{a+b+c}{b+c}\right)-\frac{b+c}{b+c}+\frac{(a+b+c)}{a+c}-\frac{a+c}{a+c}+\frac{(a+b+c)}{a+b}-\frac{(a+b)}{a+b}$$

$$=(a+b+c)\left[\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a+b}\right]-3$$

$$=\frac{1}{2}[2a+2b+2c]\left[\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a+b}\right]-3$$

$$=\frac{1}{2}\left[(b+c)+(a+c)+(a+b)\right]\left[\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a+b}\right]-3$$

$$\geq \frac{1}{2}\times 9 \text{ using (1)} \quad -3$$

$$=\frac{3}{2}.$$

Suggested solutions	Marker's Feedback

b i) LHS $= a^2 - ab + b^2$ (See alternate solution on p30)

$= \frac{1}{4}(a+b)^2 + \frac{3}{4}(a-b)^2$ where $\frac{3}{4}(a-b)^2 \geq 0$ ①

$\geq \left(\frac{a+b}{2}\right)^2$

See other alternate solution on p.

ii) Cos-e rule

$c^2 = a^2 + b^2 - 2ab\cos\angle BCA$

If $\angle BCA \geq 60°$ then $\cos\angle BCA \leq \frac{1}{2}$

$c^2 \geq a^2 + b^2 - 2ab \times \frac{1}{2}$ ①

$c^2 \geq a^2 - ab + b^2$ (1)

with equality iff $\angle BCA = 60°$

Most students could get to $c^2 \geq a^2 - ab + b^2$ but could not get any further.

Then $c \geq \sqrt{a^2 - ab + b^2} \geq \frac{a+b}{2}$ from (i) (2) ①

with equality iff $a = b$ and $\angle BCA = 60°$

Some thought as $\angle ACB \geq 60°$, $c > a$ or b which ISN'T true [what if $\angle BAC = 80°$?]

They did not realise to use Part (i) to show $c^2 \geq \left(\frac{a+b}{2}\right)^2$ $\therefore 2c \geq a+b$.

$c^3 \geq \left(\frac{a+b}{2}\right)(a^2 - ab + b^2)$

$= \frac{a^3 + b^3}{2}$

$2c^3 \geq a^3 + b^3$ with equality iff ΔABC is equilateral ①

There were no marks awarded for just stating the equality case when $\angle BCA = 60°$.

Suggested solutions	Marker's Feedback

Q11 b) i) alternative solution

$$R.H.S = \left(\frac{a+b}{2}\right)^2$$

$$= \frac{a^2}{4} + \frac{b^2}{4} + \frac{2ab}{4}$$

$$\leq \frac{a^2}{4} + \frac{b^2}{4} + \frac{a^2+b^2}{4} \quad \text{as } a^2+b^2 \geq 2ab$$

$$= \frac{a^2+b^2}{2}$$

$$= \frac{a^2+b^2-a^2+b^2}{2}$$

$$\leq \frac{a^2+b^2-2ab}{2} \quad (a^2+b^2 \geq 2ab)$$

$$= a^2+b^2-ab$$

$$= L.H.S$$

$$R.H.S \leq L.H.S$$

Mathematics Assessment Solutions

Girraween High School

page

❑ Year 11	❑ Extension 1	Year 2026
❑ Year 12	❑ Extension 2	Task Number

Suggested solutions | **Marker's Feedback**

(11)(b)(i) Alternate solution was to show $(a^2-ab+b^2)-\left(\frac{a+b}{2}\right)^2 \geq 0$

Proof:

$$(a^2-ab+b^2)-\left(\frac{a+b}{2}\right)^2$$

$$=(a^2-ab+b^2)-\left[\frac{a^2}{4}+\frac{ab}{2}+\frac{b^2}{4}\right]$$

$$=\frac{3}{4}a^2-\frac{3}{2}ab+\frac{3}{4}b^2$$

$$=\frac{3}{4}\left[a^2-2ab+b^2\right)$$

$$=\frac{3}{4}(a-b)^2$$

≥ 0 as $(a-b)^2 \geq 0 \ \forall a,b \in R$.